Wave Optics

Nature of Light:

Humans have always been interested to know what light is. In the early days, a light beam was thought to consist of particles. Later, the phenomena of interference and diffraction were demonstrated which could be explained only by assuming a wave model of light. Much later, it was shown that phenomena such as the photoelectric effect and the Compton Effect could be explained only if we assume a particle model of light. Now, as we know, the values of the mass and charge of electrons, protons, alpha particles, etc., are known to a tremendous degree of accuracy approximately one part in a billion! Their velocities can also be changed by the application of electric and magnetic fields. Thus, we usually tend to visualize them as tiny particles. However, they also exhibit diffraction and other effects which can be explained only if we assume them to be waves. Thus, the answers to the questions such as “What is an electron” or “What is light?” are very difficult. Indeed electrons, protons, neutrons, photons, alpha particles, etc., are neither particles nor waves. The modern quantum theory describes them in a very abstract way which cannot be connected with everyday experience.

Corpuscular theory: Rectilinear propagation of light is a natural deduction on the basis of corpuscular theory. This theory can also explain reflection and refraction, though the theory does not clearly envisage why how and when the force of attraction or repulsion is experienced perpendicular to the reflecting or refracting surface by a corpuscle. Newton assumed that the corpuscles possess fits which allow them easy reflection at one stage and easy transmission at the other. According to Newton’s corpuscular theory the velocity of light in a denser medium is higher than the velocity in a rarer medium. But the experimental results of Foucault and Michelson show that the velocity of light in a rarer medium is higher than that in a denser medium. Interference could not be explained on the basis of corpuscular theory because two material particles cannot cancel one another's effect. The phenomenon of diffraction vrz., bending of light round corners or illumination of geometrical shadow cannot be conceived according to corpuscular theory because a corpuscle travelling at high speed will not be deviated from its straight line path. Certain crystals like quartz, calcite etc. exhibit the phenomenon of double refraction. Explanation of this has not been possible with the corpuscle concept. The unsymmetrical behavior of light about the axis of propagation (viz. polarization of light) cannot be accounted for by the corpuscular theory.

Wave theory: Huygens wave theory could explain satisfactorily the phenomena of reflection and refraction. Applying the principle of secondary wave points, rectilinear propagation of light can be correlated. The phenomenon of interference can also be understood considering that light energy is propagated in the form of waves. Two wave trains of equal frequency and arnplitude and differing in phase can annul one another's effect and produce darkness. Similar to sound waves, bending of waves round obstacles is possible, thus enabling the understanding of the phenomenon of diffraction. Double refraction can also be explained on the basis of wave theory. According to Huygens, propagation of light is in the form of longitudinal waves. But in the case of longitudinal waves, one cannot expect the unsymmetrical behaviour of a beam of light about the axis of propagation. This difficulty was overcome when Fresnel suggested that the light waves are transverse and not longitudinal. On the basis of this concept, the phenomena of polarization can also be understood. Finally, on the basis of wave theory it can be shown mathematically, that the velocity of light in a rare medium is higher than the velocity of light in a denser medium. This is in accordance with the experimental results on the velocity of light.

Conclusio:. The controversy between the corpuscular theory and the wave theory existed till about the end of the eighteenth century. At one time the corpuscular theory held the ground and at another time the wave theory was accepted, the discovery of the phenomenon of interference by Thomas Young in l800, the experimental results of Foucault and Michelson on the velocity on light in different media and the revolutionary hypothesis of Fresnel in 1816 that the vibration of the ether particles is transverse and not longitudinal gave, in a way, a solid ground to the wave theory.

The next important advance in the nature of light was due to the work of Clerk Maxwell. Maxwell's electromagnetic theory of light lends support to Huygens wave theory whereas quantum theory strengthens the particle concept. It is very interesting to note, that light is regarded as a wave motion at one time and as a particle phenomenon at another time

HUYGEN’S PRINCIPLE

Huygens’ theory is essentially based on a geometrical construction which allows us to determine the shape of the wave front at any time, if the shape of the wave front at an earlier time is known. A wave front is the locus of the points which are in the same phase; for example, if we drop a small stone in a calm pool of water, circular ripples spread out from the point of impact, each point on the circumference of the circle (whose center is at the point of impact) oscillates with the same amplitude and same phase, and thus we have a circular wave front. On the other hand, if we have a point source emanating waves in a uniform isotropic medium, the locus of points which have the same amplitude and are in the same phase is spheres. In this case we have spherical wave fronts, as shown in Fig. 1(a). At large distances from the source, a small portion of the sphere can be considered as a plane, and we have what is known as a plane wave [see Fig.1(b)].

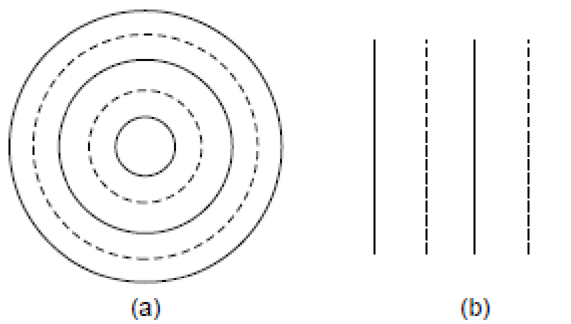


Fig.1 a) A point source emitting spherical waves. b) At large distances, a small portion of the spherical wave front can be approximated to a plane wave front, thus resulting in plane waves.

Now, according to Huygen's principle, each point of a wave front is a source of secondary disturbance, and the wavelets emanating from these points spread out in all directions with the speed of the wave. The envelope of these wavelets gives the shape of the new wave front. In Fig. 2, S_1S_2 represents the shape of the wave front (emanating from the point O) at a particular time which we denote as $t = 0$. The medium is assumed to be homogeneous and isotropic; i.e., the medium is characterized by the same property at all points, and the speed of propagation of the wave is the same in all directions. Let us suppose we want to determine the shape of the wave front after a time interval of ∇t. Then with each point on the wave front as center, we draw spheres of radius $v\nabla t$, where v is the speed of the wave in that medium. If we draw a common tangent to all these spheres, then we obtain the envelope which is again a sphere centered at O. Thus the shape of the wave front at a later time ∇t is the sphere $S'_1S'_2$.

There is, however, one drawback with the above model, because we also obtain a back wave which is not present in practice. This back wave is shown as $S'_1S'_2$ in Fig.2. In Huygen's theory, the presence of the back wave is avoided by assuming that the amplitude of the secondary wavelets is not uniform in all directions; it is maximum in the forward direction and zero in the backward direction. The absence of the back wave is really justified through the more rigorous wave theory.

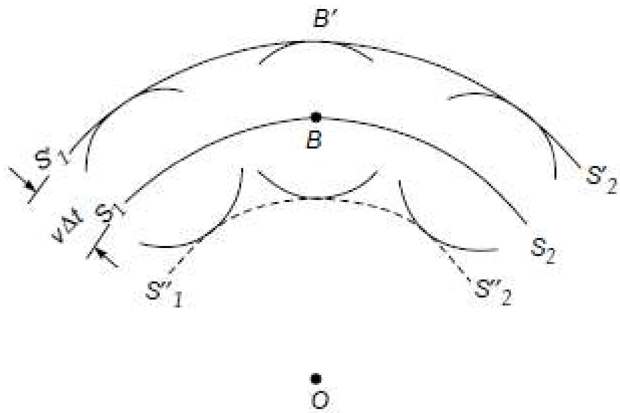


Figure 2: Huygens' construction for the determination of the shape of the wave front, given the shape of the wave front at an earlier time. S1S2 is a spherical wave front centered at O at a time, say, t = 0. S'_1 S'_2 corresponds to the state of the wave front at a time ∇t which is again spherical and centered at O. The dashed curve represents the back wave

Wavefront:

A wavefront is defined as a locus of all the particles of a medium vibrating in the same phase at a given instant. The shape of the source of disturbance decides the shape of a wavefront. The wavefront can be spherical, cylindrical or plane. The properties of wave-front are as;

Properties:

1) Energy of light flows normal to the wavefronts.
2) Time taken by light to travel from one position of wavefront to another position of wavefront is same along any ray.
3) Spacing between a pair of wavefront along any ray is constant.
4) The phase difference between any two points situated on the same wavefront is zero.
5) The line drawn perpendicular to the plane of wavefront gives the direction of propagation of wave and is called a ray of light.
6) A spherical wavefront appears as plane wavefront after travelling a large distance from the point source
7) The shape of wavefront depends upon the shape of the source of disturbance
8) At a given instant, all the particles of a medium vibrates in the same phase

Interference

The wave nature of light was demonstrated convincingly for the first time in 1801 by Thomas Young by a wonderfully simple experiment. He let a ray of sunlight into a dark room, placed a dark screen ~~in front of~~ it, pierced with two small pinholes and beyond this, at some distance a white screen. He then saw two darkish lines at both sides of a bright line, which gave ~~himself~~ sufficient encouragement to repeat the experiment, this time with spirit flame as light source, with a little salt in it, to produce the bright yellow sodium light. This time he saw a number of dark lines, regularly spaced; the first clear proof that light added to light can produce darkness. This phenomenon is called interference. Thomas Young had expected it because he believed in the wave theory of light.

Incase of sound waves, the interference pattern can be observed without much difficulty because the two interfering waves maintain a constant phase relationship. This is also the case for microwaves. However for light waves, due to the very process of emission, one cannot observe interference between the waves from two independent sources (it is difficult to observe the interference pattern even with two laser beams unless they are phase locked) although the interference takes place. Thus one tries to derive interfering waves from a single wave so that the phase relationship is maintained. The methods to achieve this can be classified under two broad categories. Under the first category, in a typical arrangement a beam is allowed to fall on two closely spaced holes and the two beams emanating from the holes interfere. This method is known as division of wavefront. In the other method, known as division of amplitude, a beam is divided at two or more reflecting surfaces and the reflected beams interfere. Therefore both the methods are based on one underlying principle, namely the superposition principle. This reveals that interference is an important consequence of superposition of coherent waves.

Definition

If two or more light waves of the same frequency overlap at a point, the resultant effect depends on the phases and of the waves as well as their amplitudes. The resultant wave at any point at any instant of time is governed by the principle of superposition. The combined effect at each point of the region of superposition is obtained by adding algebraically the amplitudes of the individual waves. Let us assume here that the component waves are of same amplitude.

At certain points, the two waves may be in phase. The amplitude of the resultant wave will then be equal to the sum of the amplitudes of the two waves, as shown in fig 1 (a). Thus the amplitude of the resultant wave is

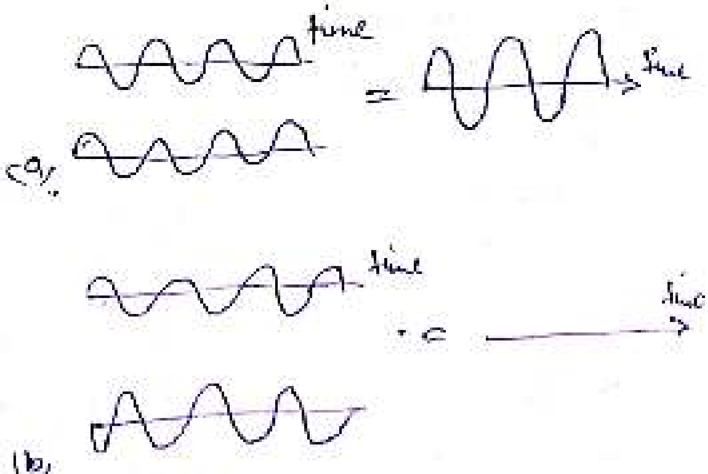


fig (1)

$$A_R = A + A = 2A \quad \text{— (1)}$$

Hence the intensity of the resultant wave is

$$I_R \propto A_R^2 = 2^2 A^2 = 2^2 I \quad — (2)$$

It is obvious that the resultant intensity is greater than the sum of the intensities due to individual waves.

$$I_R > I + I = 2I \quad — (3)$$

Therefore the interference produced at these points is known as constructive interference. A stationary bright band of light is observed at points of constructive interference.

At certain other points, the two waves may be in opposite phase. The amplitude of the resultant wave will be equal to the sum of the two waves as shown in fig 1(b). Thus the amplitude of the resultant wave is

$$A_R = A - A = 0 \quad — (4)$$

Hence the intensity of the resultant wave is

$$I_R \propto 0^2 = 0 \quad — (5)$$

It is obvious that the resultant intensity is less than the sum of the intensities of due to individual waves.

$$I_R < 2I.$$

Therefore the interference produced at these points is known as destructive interference. A stationary dark band of light is observed at points of destructive interference. Thus we see that redistribution of energy took place in the region.

Thus, when two or more coherent waves of light are superimposed, the resultant effect is brightness in certain regions and darkness at other regions. The regions of brightness and darkness alternate and may take the form of straight bands, or circular rings, or any other complex shape. The alternate dark and bright bands are called interference fringes. The phenomenon of redistribution of ~~energy~~ light energy due to the superposition of light waves from two or more coherent sources is known as interference.

Whether the condition of =n (2) or =n (4) occurs at a point is solely determined by the difference in the optical paths traversed by the waves that are superposing at that point.

Generally it is difficult to observe a stationary interference pattern as far as light waves are concerned. For ~~ex~~ example if we use ~~to~~ two conventional light sources (like two sodium lamps) illuminating two pinholes, we will not observe any interference pattern on the screen. This can be understood from the following reasoning. In a conventional light source, light comes from a large number of independent atoms, each atom emitting light for about 10^{-10} seconds i.e light emitted by an atom is essentially a pulse lasting for only 10^{-10} seconds. Even if the atoms were emitting under similar conditions, waves from different atoms would differ in their initial phase.

Consequently, light coming out from the holes S_1 and S_2 (fig 2) will have a fixed phase relationship for a period of about 10^{-10} seconds, hence the interference pattern will keep on changing every billionth of a second. The eye can notice intensity changes which last at least for a tenth of a second and hence we will observe a uniform intensity over the screen. However, if we have a camera whose time of shutter opening can be made less than 10^{-10} seconds, then the film will record an interference pattern. We summarise the above results by noting that light beams from two independent sources do not have any fixed phase relationship, as such they do not produce any stationary interference pattern.

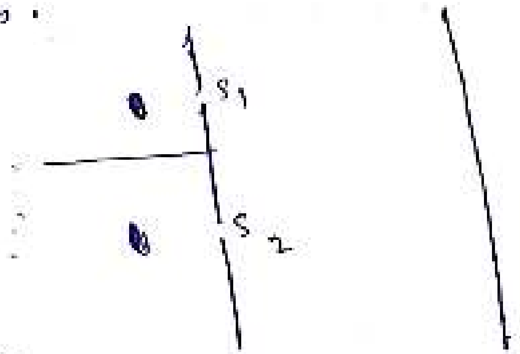
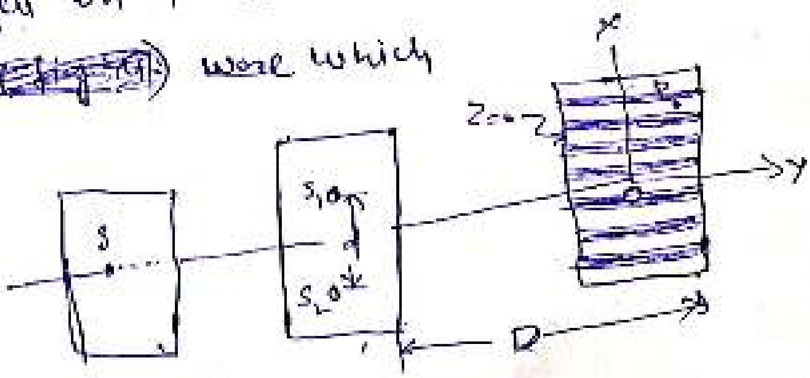


Fig 2.1

Thomas Young in 1801 devised an ingenious but simple method to lock the phase relationship between the two sources. The trick lies in the division of a single wavefront into two, these two split wavefronts act as if they emanated from two sources having a fixed phase relationship and therefore, when these two waves were allowed to interfere a stationary interference pattern was obtained. In the actual experiment, a light source illuminates the pinhole S_0 (fig 3). Light diverging from this pinhole fell on a barrier which contained two pinholes S_1 and S_2 ~~(fig 4)~~ were which were very close to one another and were located equidistant from S.



(Fig 3)

Spherical waves emanating from S_1 and S_2 (fig 4) were coherent and on the screen beautiful fringes were obtained. In order to show that this was indeed an interference effect, Young showed that the fringes on the screen disappear when S_1 (or S_2) is covered up. Young explained the interference pattern by considering the principle of superposition and by measuring the distance between the fringes he calculated the wavelength. Fig 4 shows the section of the wavefront on the plane containing S, S_1 and S_2 (which is the x-z plane).

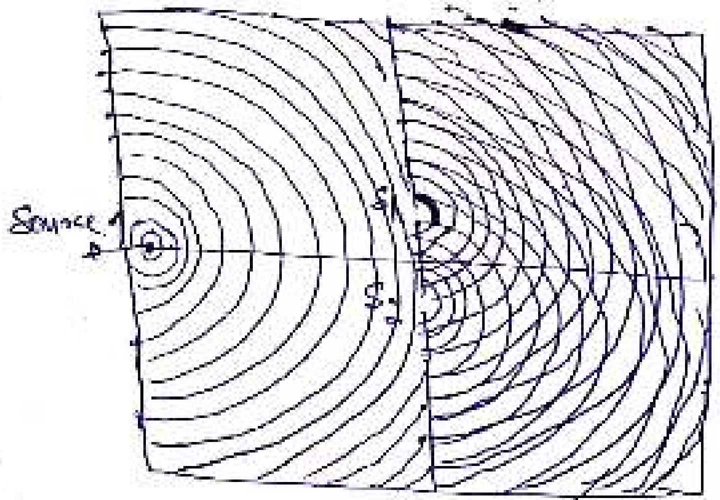


Fig 4.

Theoretical concept of interference:

Let us consider two sources of light S_1 and S_2 as shown in fig 5. Let us assume that the sources are identical and produce harmonic waves of same wavelength and that the waves are in the same phase at S_1 and S_2. Light from these sources travel

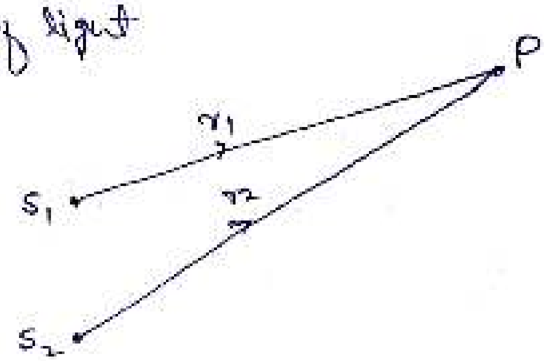


Fig 5

along differend paths. S_1P and S_2P and ~~meet~~ meet at a point P.

Let us assume that the electric field components of the two waves arriving at point P vary with time as:

$$E_A = E_1 \sin \omega t \quad — (1)$$

and $$E_B = E_2 \sin(\omega t + \delta) \quad — (2)$$

where δ is the phase difference between them.

According to young's principle of superposition, the resultant electric field at the point P due to the simultaneous action of the two waves is given by

$$E_R = E_A + E_B \quad — (3)$$

$$= E_1 \sin \omega t + E_2 \sin(\omega t + \delta)$$

$$= E_1 \sin \omega t + E_2 (\sin \omega t \cdot \cos\delta + \sin\delta \cos \omega t)$$

$$= (E_1 \cancel{\sin} + E_2 \cos\delta) \sin \omega t + E_2 \sin\delta \cos \omega t \cdot \quad — (4)$$

= n(4) shows that the superposition of two Sinusoidal waves having the same frequency but with a different phase difference produces a sinusoidal wave with same frequency but with a different amplitude E.

Let $E_1 + E_2 \cos\delta = E \cos\phi \quad — (5)$

and $E_2 \sin\delta = E \sin\phi \quad — (6)$

where E is the amplitude of the resultant wave. and ϕ is the new initial phase angle. In order to solve for E and ϕ, we square the eqn (5) & (6) and add them.

$$(E_1 + E_2\cos\delta)^2 + E_2^2\sin^2\delta = E^2(\cos^2\phi + \sin^2\phi)$$

or $$E^2 = E_1^2 + E_2^2\cos^2\delta + 2E_1E_2\cos\delta + E_2^2\sin^2\delta$$

$$= E_1^2 + E_2^2(\cos^2\delta + \sin^2\delta) + 2E_1E_2\cos\delta$$

$$E_0^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\delta \quad \text{—— (7)}$$

Thus it is seen that the square of the amplitude of the resultant wave is not a simple sum of the squares of the amplitudes of the superposing waves, there is an additional term which is known as the interference term.

YOUNG'S DOUBLE SLIT EXPERIMENT – WAVEFRONT DIVISION

As early as in 1665 Grimaldi attempted to produce interference between two beams of light. He directed sunlight into a dark room through two pinholes in a screen, with an expectation that bright and dark bands would be observed in the area where the beams overlap on each other. He observed uniform illumination instead. In 1801, about one hundred thirty years later, Thomas Young gave the first demonstration of the interference of light waves. Young admitted the sunlight through a single pinhole and then directed the emerging light onto two pinholes. Finally the light was received on a screen. The spherical waves emerging from the pinholes interferred with each other and a few coloured fringes were observed on the screen. The amount of light that emerged from the pinhole was very small and the fringes were faint and difficult to observe.

The pinholes were later replaced with narrow slits that let through much more light. The sunlight was replaced by monochromatic light. Young's experiment is known as double-slit experiment.

Fig. 6. shows a plan view of the basic arrangement of the double slit experiment.

The primary light source is a monochromatic source, it is generally a sodium lamp, which emits yellow light of wavelength at around 5893 Å. This light is not suitable for causing interference because emissions from different parts of any

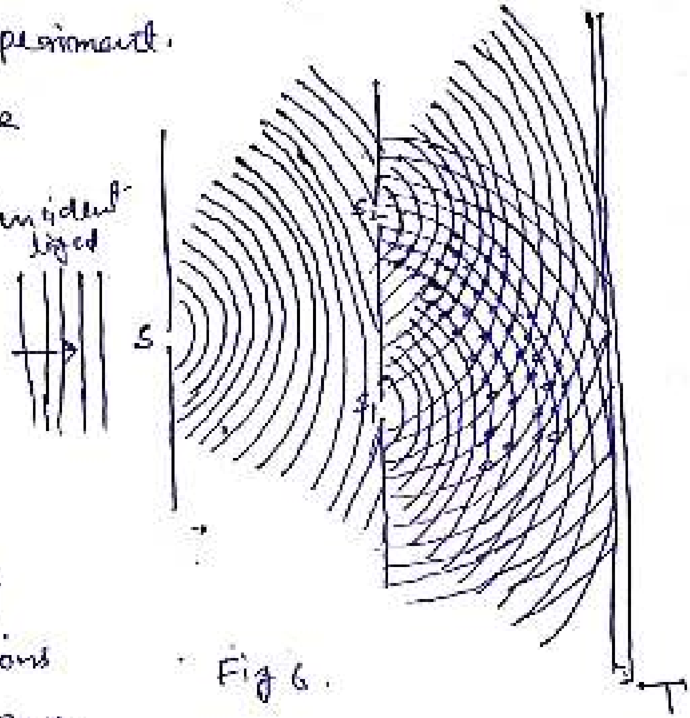


Fig 6.

ordinary source are not coherent. Therefore, the monochromatic light is allowed to pass through a narrow slit at S. The light coming out of the slit originated from only a small region of light source and hence behaves more nearly like an ideal light source. Cylindrical wavefronts are produced from the slit S, the primary light source, which fall on the two narrow closely spaced slits, S_1 and S_2 as shown in fig 6. The ~~slits~~ slits at S_1 and S_2 are very narrow. The cylindrical waves emerging from the slits overlap. If the slits are equidistant from S, the phase of the wave at S_1 will be the same as the phase of the wave at S_2. Further, waves leaving S_1 and S_2 are therefore always in phase. Hence, sources S_1 and S_2 act as secondary coherent sources. The waves leaving from S_1 and S_2 interfere and produce alternate bright and dark bands on the screen at T

OPTICAL PATH DIFFERENCE BETWEEN THE WAVES AT P.

Let P be an arbitrary point on screen T, which is at a distance D from the double slits. Let θ be the angle between MP and the horizontal line MO. Let S_1N be a normal on to the line S_2P. The distances PS_1 and PN are equal. The waves emitted at the slits, S_1 and S_2 are initially in phase with each other.

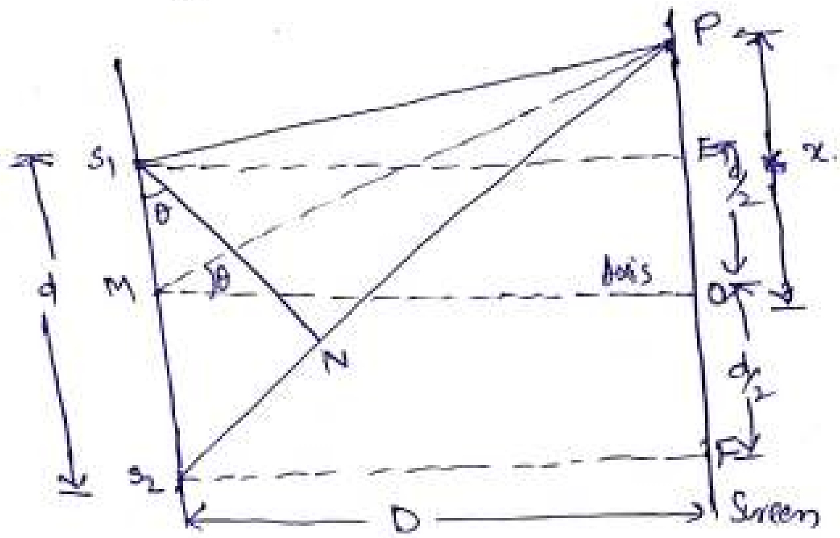


Fig 7.

The difference in the path lengths of these two waves is S_2N. We assume that the experiment is carried out in air. Therefore the optical paths are identical with geometrical paths. The nature of the interference of the two waves at P depends simply on how many waves are contained in the length of the path difference S_2N. If S_2N contains an integral number of

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wavelengths, the two waves interfere constructively producing a maximum in the intensity of light on the screen at P. If it contains an odd number of half-wavelengths, then the two waves interfere destructively and produce a minimum intensity at P.

Let the point P be at a distance x from O (Fig 7). Then

$$PE = x - d/2 \text{ and } PF = x + d/2$$

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + (x+d/2)^2\right] - \left[D^2 + (x-d/2)^2\right]$$

$$(S_2P)^2 - (S_1P)^2 = 2xd.$$

$$S_2P - S_1P = \frac{2xd}{S_2P + S_1P}$$

We can approximate that $S_2P \cong S_1P \cong D$

$\therefore$ path difference $= S_2P - S_1P = \frac{2xd}{2D} = \frac{xd}{D}$

We now find out the conditions for observing bright and dark fringes on the screen.

CONDITIONS FOR INTERFERENCE

We may now summarise the conditions that are to be fulfilled in order to observe a distinct well-defined interference pattern.

(A) Conditions for sustained interference

i) The waves from the two sources must be of the same frequency.

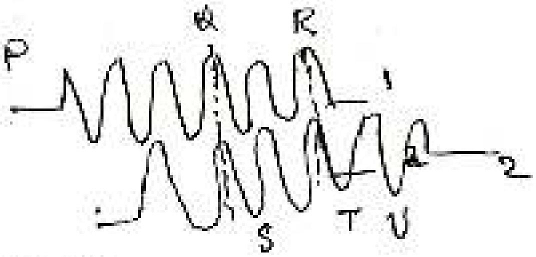
of the light waves differ in frequency, the phase difference fluctuates irregularly with time. consequently, the intensity at any point fluctuates with time and we will not observe steady interference.

(ii) The two light waves ~~are~~ must be coherent.

If the light waves are coherent, then they maintain a fixed phase difference over a time and space. Hence, a stationary interference pattern will be observed.

(iii) The path difference between the overlapping waves must be less than the coherence lengths of the waves.

We have already learnt that light is emitted in the form of wave trains and a finite coherence length characterizes them. If we consider two interfering wave trains, having constant phase difference as in figure Q, the ~~difference~~ interference effects occur due to parts QR of wave 1 and ST of wave 2.



For the parts PQ and TU interference will not occur. Therefore, the interference pattern does not appear distinctly. When the entire wave train PR overlaps on the wave train SU, interference patterns will be distinct. On the other hand, when the ratio difference between the waves 1 and 2 becomes very large, the wave trains arrive at different times and do not overlap on each other. Therefore, in such cases interference does not take place. The interference pattern completely vanishes if the path difference is equal to the coherence length. It is hence required that

$$0 < L_{coh}.$$

(iv) If the two sets of waves are plane polarized, their planes of polarization must be the same. waves polarized in perpendicular planes cannot produce interference effects.

(B) Conditions for formation of distinct fringe pattern:

(v) The two coherent sources must lie close to each other in order to discern the fringe pattern. If the sources are far apart, the fringe width will be very small and fringes are not seen separately.

(vi) The distance of the screen from the two sources must be large.

(vii) The vector sum of the overlapping electric field vectors should be zero in the dark regions for obtaining distinct bright and dark fringes. The sum will be zero only if the vectors are anti-parallel and have the same magnitude.

FRESNEL BIPRISM :-

Fresnel used a biprism to show interference phenomenon. The biprism consists of two prisms of very small refracting angles joined to base. In practice, a thin glass plate is taken and one of its faces is ground and polished till a prism is formed with an obtuse angle of about 179° and two side angles of the order of 30'.

When a light ray is incident on an ordinary prism, the ray through an angle called the angle of deviation. As a result, the ray emerging out of the prism appears to have emanated from a source S' located at a small distance above real source, as shown in fig 8(b) we say that the prism produced a

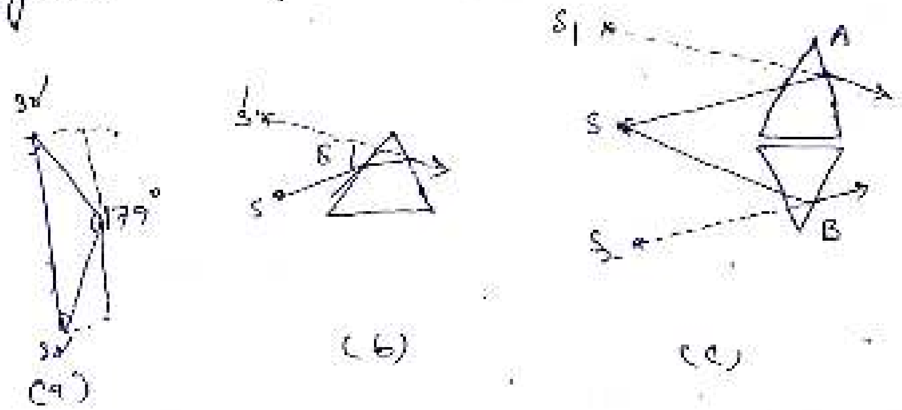


Fig 8.

virtual image of the source. A biprism, in the same way, creates two virtual sources S_1 and S_2, as seen in fig 8(c). These two virtual sources are images of the same source S produced by refraction and are hence coherent.

Experimental Arrangement:-

The biprism is mounted suitably on an optical
bench. An optical bench consists of two horizontal
long rods, which are taken strictly parallel to each
other and at the same ~~time~~ level. The rods carry
uprights on which the optical components are positioned.
A monochromatic light source such as sodium
vapor lamp illuminates a vertical slit s. Therefore,
the slit s acts as a narrow linear monochromatic
light source. The biprism is placed in such a way
that its refracting edge is parallel to the length of
the slit S. A single cylindrical wavefront impinges on both
prisms. The top portion of wavefront is refracted downwards
and appear to have emanated from the virtual image
S1. The lower segment, falling on the lower part of

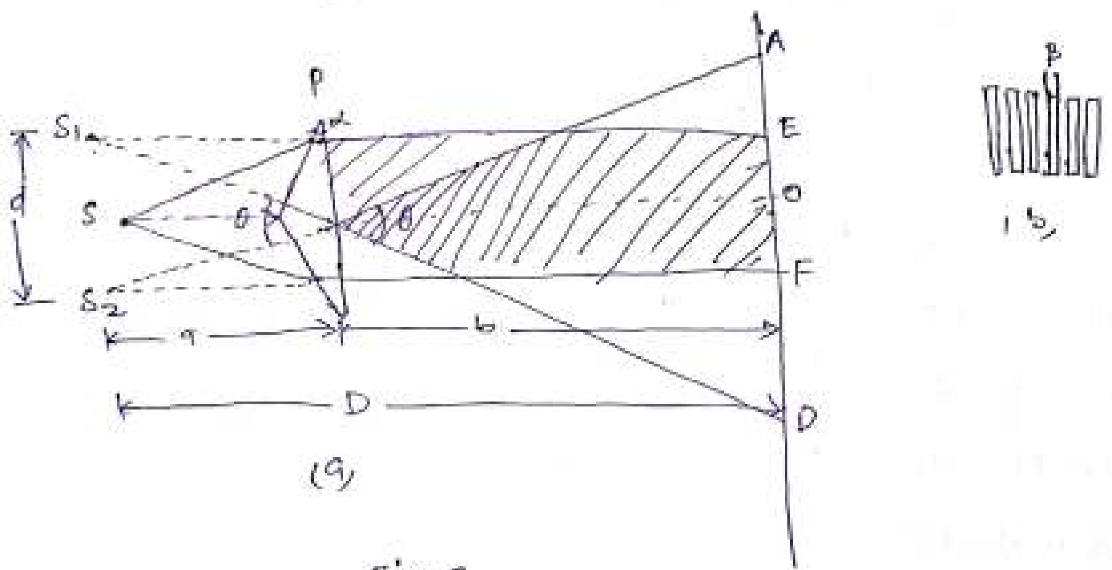


Fig 9.

~~the~~ the biprism, is refracted upward and appear to have
emanated from the virtual source S2. The virtual
sources S1 and S2 are coherent (fig 9), and hence

the light waves are in a position to interfere in the region beyond the biprism. If a screen is held there, interference fringes are seen. In order to ~~Same~~ observe fringes, a micrometer eyepiece is used.

Theory:

The theory of the interference and fringe formation in case of Fresnel biprism is the same as described for Young's double slit experiment. As the point O is equidistant from S_1 and S_2, the central bright fringe of maximum intensity occurs there. On both sides of O, alternate bright and dark fringes, as shown in fig 9(b) are produced. The width of the dark or bright fringe is given by as

$$\beta = \frac{\lambda D}{d} \quad \text{--- (1)}$$

where $D(=a+b)$ is the distance of the sources from the eyepiece.

Determination of wavelength of Light:

The wavelength of light can be determined using the equation (1). For using the relation the values of β, D and d are to be measured. These measurements are done as follows;

Adjustments:

A narrow adjustable slit S, the biprism and a micrometer eyepiece are mounted on the uprights and are ~~ag~~ adjusted to be at the same height and in a straight line. The slit is made vertical and parallel to the refracting edge of the biprism by rotating it in its own plane. It is illuminated with the light from the monochromatic source. The biprism is moved along the optical bench till, on looking through it along the axis of the optical bench, two equally bright vertical slit images are seen. Then the eyepiece is moved till the fringes appear in the focal plane of the eyepiece.

(I) Determination of fringe width:

When the fringes are observed in the field view of the eyepiece, the vertical cross wires is made to coicide with the centre of ~~the~~ one of the bright fringes. The position of the eyepiece is read on the scale, say x_0. The micrometer screw of the eye piece is moved slowly and the number of the bright fringes N that pass across the cross-wire is counted. The position of the cross wire is again read say x_N. The fringe width is the given by.

$$\beta = \frac{x_N - x_0}{N} \qquad \text{—(2)}$$

(II) Determination of d;

A convex lense of short focal length is placed between the slit and the eyepiece without disturbing their positions. The lense is moved back and forth near the biprism till a sharp pair of images of the slit is obtained in the field view of the eyepiece. The distance between the images is measured. Let it be denoted by d_1.

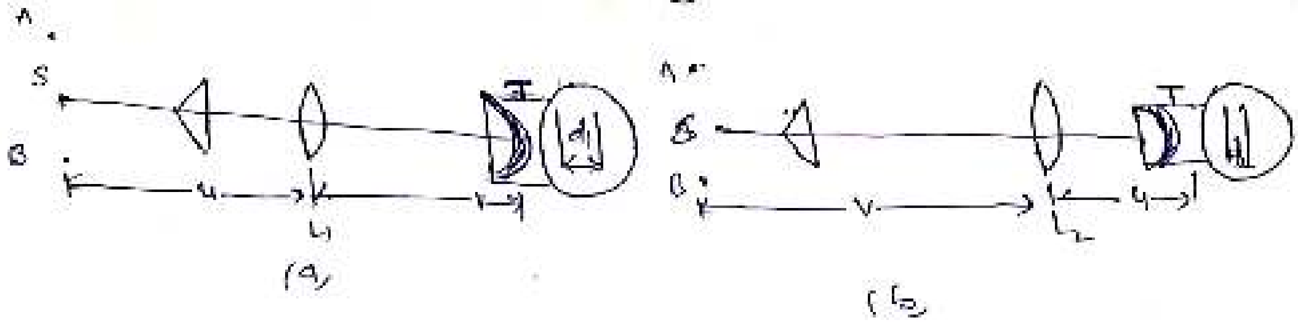


fig 10.

If u is the distance of the slit and v that of the eyepiece from the lense (fig 10a), then the magnification is

$$\frac{v}{u} = \frac{d_1}{d} \quad \text{--- (3)}$$

The lens is then moved to a position near to the eyepiece, where again a pair of images of the slit is seen. The distance between the two sharp images is again measured. Let it be d_2. Again magnification is given by

$$\frac{u}{v} = \frac{d_2}{d} \quad \text{--- (4)}$$

Note that the magnification in one position is the reciprocal of the magnification in the other position.

Multiplying the eqns (3) & (4), we obtain.

$$\frac{v}{u} \times \frac{u}{v} = \frac{d_1 d_2}{d^2} \Rightarrow \frac{d_1 d_2}{d^2} = 1$$

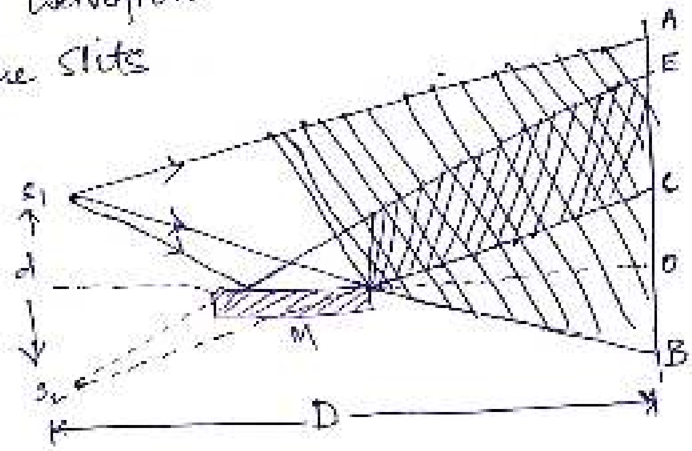
$$\Rightarrow d = \sqrt{d_1 d_2} \quad \text{--- (5)}$$

Using the values of β, d and D in the eqn (1), the wavelength λ can be computed.

(17) Lloyd's SINGLE MIRROR :-

In 1834, Lloyd devised an interesting method of producing interference, using a single mirror and using almost grazing incidence. The Lloyd's mirror consists of a plane mirror about 30cm in length and 6 to 8 cm in breadth as shown in figure. It is polished on the front surface and blackened at the back to avoid multiple reflections. A cylindrical wavefront coming from a narrow slit S_1 falls on the mirror which reflects a portion of the incident wavefront, giving rise to a virtual image of the slit S_2.

Another portion of the wavefront proceeds directly from the slits S_1 and to the screen.



The slits S_1 and S_2 act as two coherent sources. interference between direct and reflected waves occurs within the region of overlapping of the two beams and fringes are produced on the screen placed at a distance D from S_1 in the shaded portion EC.

The point O is equidistant from S_1 and S_2. Therefore central (zero-order) fringe is expected to lie at O (the perpendicular bisector of S_1S_2) and it is also expected to be bright. However it is not usually seen as since the point O lies outside the region of interference (only the direct light and not the reflected light reaches O).

By moving the screen nearer to the mirror such that it comes ~~with~~ into contact with the mirror, the point O can be just brought into the region of interference. With white light the central fringe at O is expected to be white but in practice it is dark. The occurance of dark fringe can be understood taking into the consideration of the phase change of π, that light suffers when reflected from the mirror. The phase ~~angle~~ change leads to a path difference of $\lambda/2$ and hence destructive interference occurs there.

Determination of wavelength:

$$\beta = \frac{\lambda D}{d}$$

Comparison between the fringes produced by biprism and Lloyd's mirror:

1) In biprism the complete set of fringes is obtained. In Lloyd's mirror a few fringes on one side of the central fringe are obtained, the central fringe being itself invisible.
2) In biprism the central fringe is bright, whereas in case of Lloyd's mirror, it is dark.
3) The central fringe is less sharp in biprism than that of Lloyd's mirror.

3) The central fringe is less sharp in biprism than that of Lloyd's mirror.

PHASE CHANGE ON REFLECTION :-

We will now investigate the reflection of light at an interface between two media using the principle of optical reversibility. According to this principle, in the absence of any absorption, a light ray that is reflected or refracted will retrace its original path if its direction is reversed.

Consider a light ray incident on an interface of two media of refractive indices n_1 and n_2 as shown in figure 1a. Let the amplitude reflection and transmission coefficients be r_1 and t_1, respectively. Thus, if the amplitude of the incident ray is a, then the amplitudes of the reflected and refracted rays would be ar_1 and at_1, respectively.

We now reverse the rays and

We consider a ray of amplitude at_1 incident on medium 1 and a ray of amplitude ar_1 incident on medium 2 as shown in figure 1(b). The ray of amplitude at_1 will give rise to a reflected

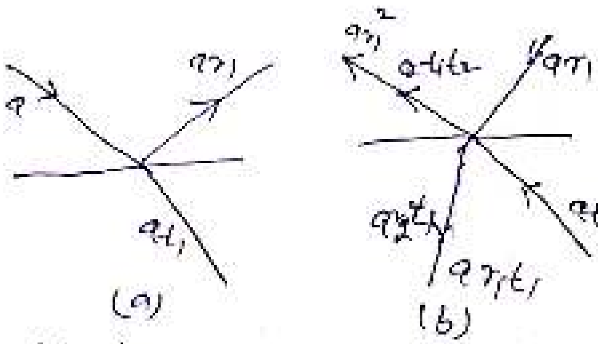


fig 1

ray of amplitude at_1r_2 and a transmitted ray of amplitude at_1t_2 where r_2 and t_2 are the amplitude reflection and transmission coefficients when a ray is incident from medium 2 on medium 1. Similarly, the ray of amplitude ar_1 will give rise to a ray of amplitude ar_1^2 and a refracted ray of amplitude ar_1t_1. According to the principle of optical reversibility the two rays of amplitudes ar_1^2 and at_1t_2 must combine to give the incident ray of fig 1(a): Thus

$$\cancel{a}r_1^2 + \cancel{a}t_1t_2 = \cancel{a}$$

$$r_1^2 + t_1t_2 = 1$$

$$t_1t_2 = 1 - r_1^2 \quad — (1)$$

Further, the two rays of amplitudes at_1r_2 and ar_1t_1 must cancel each other i.e.

$$at_1r_2 + ar_1t_1 = 0$$

$$r_2 = -r_1 \quad — (2)$$

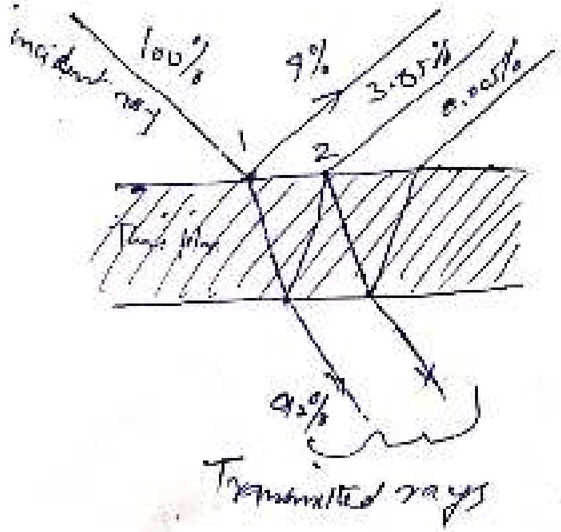
Since we know from the Lloyd's mirror experiment that an abrupt phase change of π occurs when light gets reflected by a denser medium, we may infer from eqn (2) that no such abrupt phase change occurs when light gets reflected by a rare medium. This ~~is~~ m is indeed borne out by experiments.

eqns (1) and (2) are known as Stoke's relations.

Thin Film :-

An optical medium is called a thin film when its thickness is about the order of 1 wavelength of light in visible region. Thus a film of thickness in the range 0.5 μm to 10 μm may be considered as a thin film. A thin film may be a thin sheet of transparent material such as glass, mica, an air film enclosed between two transparent plates or a soap bubble. When light is incident on such a film, a small part of it gets reflected from the top surface and a major part transmitted into the film. Again, a small part of the transmitted component is reflected back into the film by the bottom surface and the rest of it emerges out of the film. A portion of the light thus gets reflected partially several times in succession within the film (see in fig).

In transparent thin films, the two bounding surfaces strongly transmitt light and only weakly reflect the incident light. Therefore, only the first reflection at the top surface and the reflection at the bottom surface will be of appreciable strength. For example, if we consider a glass plate, having a



refractive index 1.52, the reflectivity of the top surface is given by

$$r = \left[\frac{1.52-1}{1.52+1}\right]^2 = 0.042$$

It means that about 4% of the incident light is reflected by the top surface of the glass plate, while 96% of it is transmitted into the plate. Out of the light reaching the bottom surface, again 3.8% is reflected and 92% is transmitted out the plate. Then, again out of the 3.8% of the light 0.15% is reflected at the inner boundary of the top surface and about 3.65% is transmitted out into the air. After two reflections, the intensity will become insignificantly small. At each reflection, the intensity and hence the amplitude of light wave is divided into a reflected component and a refracted component. The reflected and refracted components travel along different paths and subsequently overlap to produce interference. Therefore, the interference in thin films is called interference by division of amplitude. Newton and Robert Hooke first observed the thin film interference. However, Thomas Young gave the correct explaination of the phenomena. A thin film may be uniform or non-uniform in its structure. However, as long as its thickness lies within the specified limits, interference of light occurs.

NEWTON'S RINGS

When a Plano-convex lens of long focal length is placed on a plane glass plate, a thin film of air is enclosed between the lower surface of the lens and the upper surface of the plate. The thickness of the air film is very small at the point of contact and gradually increases from the centre outwards. The fringes produced with monochromatic light are circular. The fringes are concentric circles, uniform in thickness and with the point of contact as the centre. When viewed with white light, the fringes are colored. With monochromatic light, bright and dark circular fringes are produced in the air film.

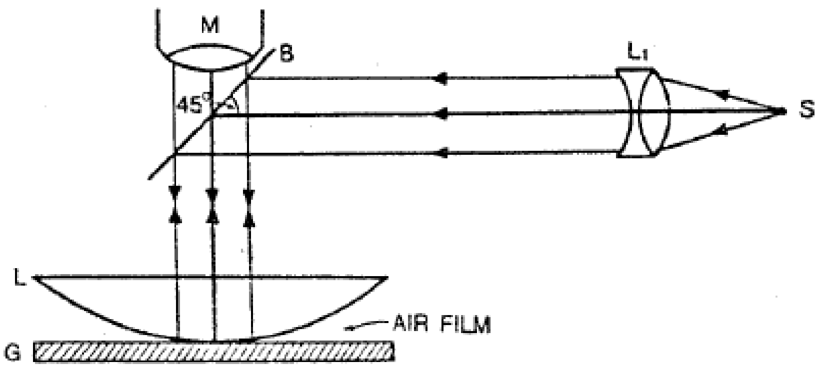


Figure 1

S is a source of monochromatic light at the focus of lens L_1 shown in figure 1. A horizontal beam of light falls on the glass plate B at 45^0. The glass plate B reflects a apart of the incident light towards the air film enclosed by the lens L and the plane glass plate G. the reflected beam from the air film is viewed with a microscope. Interference takes place and dark and bright circular fringes are produced. This is due to the interference between the light reflected from the lower surface of the lens and the upper surface of the glass plate G.

THEORY: NEWTON'S RINGS BY REFLECTED LIGHT

Suppose the radius of curvature of the lens is R and the air film is of thickness t at a distance of OQ/2 = r from the point of contact O as shown in figure 2.

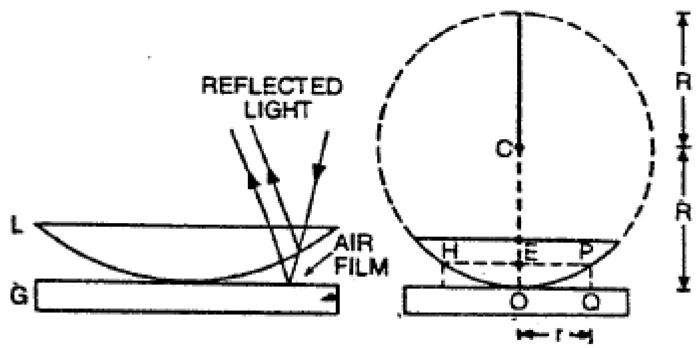


Figure 2

Here, interference is due to the reflected light. Therefore, for the bright rings we have;

$$2\mu t\cos\theta = (2n-1)\lambda/2 \qquad (1)$$

Where $n = 1, 2, 3, \ldots$ etc

Here, θ is small, therefore cosθ = 1 and for air μ = 1, we get;

$$2t = (2n-1)\lambda/2 \qquad (2)$$

For dark rings,

$$2\mu t\cos\theta = n\lambda$$

or $2t = n\lambda$

where $n = 0, 1, 2, \ldots$ etc (3)

from figure 2 we have

$$EP \times HE = OE \times (2R-OE)$$

But $EP = HE = r$, and $OE = PQ = t$

And $2R-t = 2R$ (approximately)

$$r^2 = 2R.t$$

or $t = r^2/2R$

Substituting the value of t in equations (2) and (3) we have;

For bright rings

$$r^2 = (2n-1)\lambda R/2 \qquad (4)$$

$$r = \sqrt{\frac{(2n-1)\lambda R}{2}} \qquad (5)$$

For dark rings

$$r^2 = n\lambda R \qquad (6)$$

$$r = \sqrt{n\lambda R} \qquad (7)$$

When n= 0, the radius of the dark ring is zero and the radius of the bright ring is $\sqrt{\frac{\lambda R}{2}}$. Therefore, the centre is dark. Alternately dark and bright rings are produced as shown in figure 3.

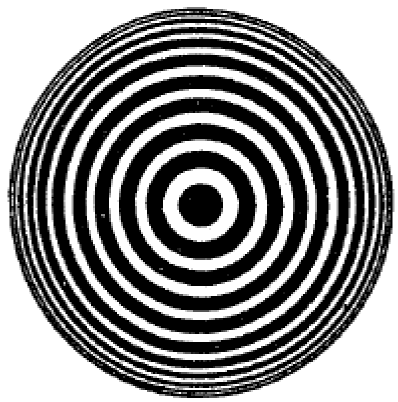


Figure 3

If D is the diameter of the dark ring, then

$$D = 2r = 2\sqrt{n\lambda R} \qquad (8)$$

For the central dark ring, n =0 which depicts D = 0. This corresponds to the centre of the Newton's rings. While counting the order of the dark rings 1, 2, 3 etc. the central ring is not counted. Therefore for the first dark ring n= 1.

$$D_1 = 2\sqrt{\lambda R}$$

For the second dark ring n=2, and $D_2 = 2\sqrt{2\lambda R}$ and for n the dark ring $D_n = 2\sqrt{n\lambda R}$. Therefore, the fringe width decreases with the order of the fringe and the fringes got closer with increase in their order. For bright rings

$$r^2 = (2n-1)\lambda R/2 \qquad (9)$$

or

$$D2 = 2(2n-1)\lambda R \qquad (10)$$

$$r_n = \sqrt{\frac{(2n-1)\lambda R}{2}} \qquad (11)$$

Therefore, the bright rings can be obtained by substituting the n = 1, 2, 3 etc directly in equation (9).

DETERMINATION OF WAVELENGTH OF SODIUM LIGHT USING NEWTON'S RINGS

The arrangement used is shown ih Fig. 1 above. S is a source of sodium light. A parallel beam of light from the lens L_1, is reflected by the glass plate B inclined at an angle of 45^o to the horizontal. L is a Plano-convex lens of large focal length. Newton's rings are viewed through B by the travelling microscope M focused on the air film. Circular bright and dark rings are seen with the centre dark. With the help of a travelling microscope, measure the diameter of the nth dark ring.

Suppose, the diameter of the nth ring = D_n

$$r^2 = n\lambda R$$

But $\quad r_n = D_n/2$

Therefore $\quad (D_n)^2/4 = n\lambda R$

$$D_n^2 = 4n\lambda R \qquad (1)$$

Measure the diameter of the n+mth dark ring, let it be D_{n+m}

Therefore $\quad (D_{n+m})^2 = 4(n+m)\lambda R \qquad (2)$

Subtracting (1) from (2) we have;

$$(D_{n+m})^2 - (D_n)^2 = 4m\lambda R$$

or $\quad \lambda = (D_{n+m})^2 - (D_n)^2/4mR$

Hence, λ can be calculated.

REFRACTIVE INDEX OF LIQUAID USING NEWTON'S RINGS

The experiment is performed when there is an air film between the Plano-convex lens and the optically plane glass plate. These are kept in a metal container C. The diameter of the nth and the (n + m)th dark rings are determined with the help of a travelling microscope (Fig. 4).

For air

$$(D_{n+m})^2 = 4(n+m)\lambda R; \qquad D_n^2 = 4n\lambda R$$

$$D^2_{n+m} - D^2_n = 4m\lambda R \qquad (1)$$

The liquid is poured in the container C without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. The diameters of the nth ring and the (n + m)th ring are determined.

For the liquid, $2\mu t\cos\theta = n\lambda$ for dark rings

or $\quad 2\mu t = n\lambda$. But , $t = r^2/2R$

or $\quad 2\mu r^2/2R = n\lambda$

or $r2 = n\lambda R/\mu$, but $r = D/2$; $D2 = 4n\lambda R/$

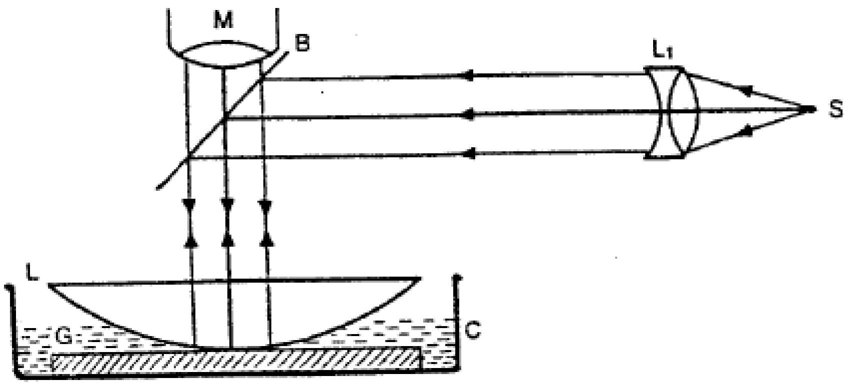


Figure 5

If D_n' is the diameter of the nth ring and D'_{n+m} is the diameter of the (n+m)th ring, then

$(D'_{n+m})^2 = 4(n+m)\lambda R/\mu$; $(D'_n)^2 = 4n\lambda R/\mu$

or $$(D'_{n+m})^2 - (D'_n)^2 = 4m\lambda R/\mu \qquad (2)$$

or $$\mu = 4m\lambda R/(D'_{n+m})^2 - (D'_n)^2 \qquad (3)$$

if m, λ, R, D'_{n+m} and D'_n are known μ can be calculated.

MICHELSON INTERFEROMETER

A schematic diagram of the Michelson interferometer is shown in Fig. 1; *S* represents a light source (which may be a sodium lamp) and *L* represents a ground glass plate so that an extended source of almost uniform intensity is formed. G_1 is a beam splitter; i.e., a beam incident on G_1 gets partially reflected and partially transmitted. M_1 and M_2 are good-quality plane mirrors having very high reflectivity. One of the mirrors (usually M_2) is fixed and the other (usually M_1) is capable of moving away from or toward the glass plate G_1 along an accurately machined track by means of a screw. In the normal adjustment of the interferometer, mirrors M_1 and M_2 are perpendicular to each other and G_1 is at 45^0 to the mirror.

Waves emanating from a point *P* get partially reflected and partially transmitted by the beam splitter G_1, and the two resulting beams are made to interfere in the following manner: The reflected wave (shown as 1 in Fig. 1) undergoes a further reflection at M_1, and this reflected wave gets (partially) transmitted through G_1; this is shown as 5 in the figure. The transmitted wave (shown as 2 in Fig. 1) gets reflected by M_2 and gets (partially) reflected by G_1 and results in the wave shown as 6 in the figure. This can be easily seen from the fact that if x_1 and x_2 are the distances of mirrors M_1 and M_2 from the plate G_1, then to the eye the waves emanating from point P will appear to get reflected by two parallel mirrors (M_1 and M_2' —see Fig. 1) separated by a distance $x_1 \sim x_2$.

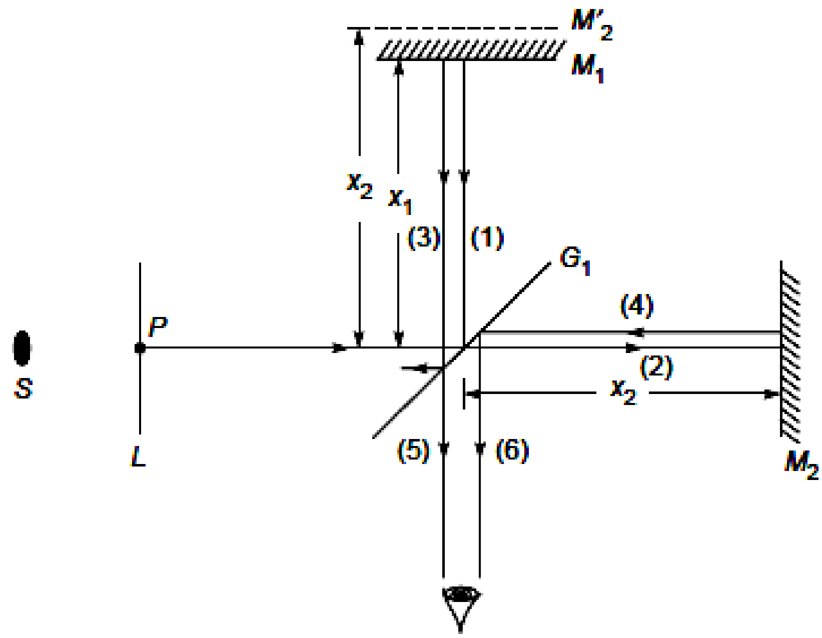


Figure 1: Schematic of Michelson Interferometer

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If we use an extended source, then no definite interference pattern will be obtained on a photographic plate placed at the position of the eye. Instead, if we have a camera focused for infinity, then on the focal plane we will obtain circular fringes, each circle corresponding to a definite value of θ (see Fig 2). Now, if the beam splitter is just a simple glass plate, the beam reflected from mirror M_2 will undergo an abrupt phase change of π (when getting reflected by the beam splitter), and since the extra path that one of the beams will traverse will be $2(x_1 \sim x_2)$, the condition for destructive interference will be

$$2d \cos \theta = m\lambda$$

where $m = 0, 1, 2, 3, \ldots$ and $d = x_1 \sim x_2$ and the angle θ represents the angle that the rays make with the axis (which is normal to the mirrors as shown in Fig. 2). Similarly, the condition for a bright ring is

$$2d \cos\theta = (m+1/2)\lambda$$

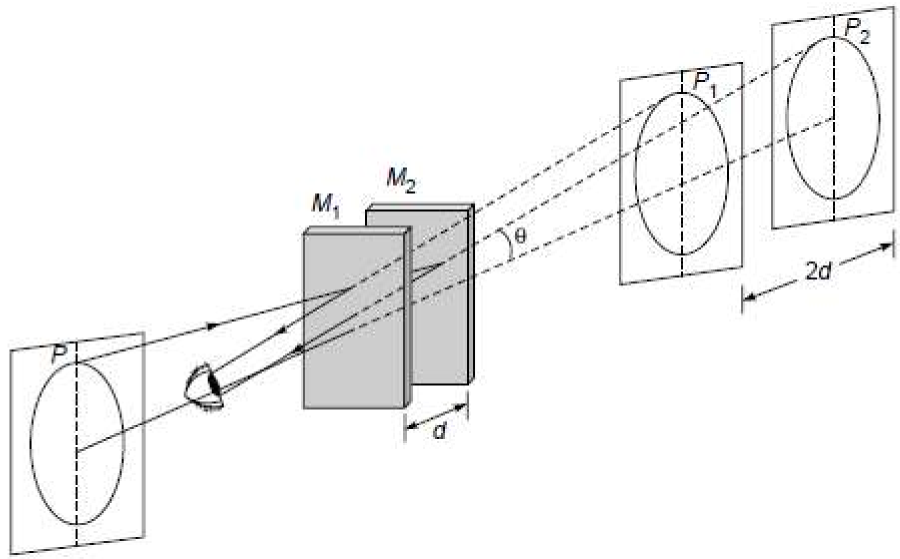

Figure 2: A schematic of the formation of circular fringes.

We start reducing the value of *d*, the fringes will appear to collapse at the center and the fringes become less closely placed Thus, as *d* decreases, the fringe pattern tends to collapse toward the center. (Conversely, if *d* is increased, the fringe pattern will expand.) Indeed, if *N* fringes collapse to the center as mirror *M*1 moves by a distance *d*0, then we must have

$$2d = m\lambda$$

$$2(d-d_0) = (m-N)\lambda$$

Where we have set $\theta' = 0$ because we are looking at the central fringe. Thus

$$\lambda = 2d_0/N$$

This provides us with a method for the measurement of the wavelength. In an actual Michelson interferometer, the beam splitter G_1 consists of a plate (which may be about 1/2 cm thick), the back surface of which is partially silvered, and the reflections occur at the back surface. It is immediately obvious that beam 5 traverses the glass plate three times, and to compensate for this additional path, one introduces a "compensating plate" G_2 which is exactly of the same thickness as G_1. The compensating plate is not really necessary for a monochromatic source because the additional path $2(n-1)t$ introduced by G_1 can be compensated by moving mirror M_1 by a distance $(n-1)t$, where n is the refractive index of the material of the glass plate G_1.

APPLICATIONS OF MICHELSON INTERFEROMETER

We can perform three types of measurements with a Michelson interferometer

i) Wavelength of light

ii) Width and fine structure of spectrum lines, and

iii) Refractive indices

a) Determination of wavelength of monochromatic light:

The interferometer is first adjusted for circular fringes. Thereafter, mirror M_2 is adjusted so as to obtain a bright spot at the centre of the field of view. If thickness of air film is d and the order of the fringe is n, then we have

$$2d\cos\theta = n\lambda$$

At the centre, $\theta = 0$ so that the above relation reduces to

$$2d = n\lambda$$

If we move from M_2 away from M_1 by $\lambda/2$, 2d increases by λ and n is replaced by (n+1), i,e the centre is now occupied by (n+1)th bright spot. In fact each time , M2 moves through $\lambda/2$, the next bright spot appears at the centre. If p new fringes appear at the centre of the field when M2 moves through a distance x, we can write

$$x = p\lambda/2$$

so that

$$\lambda = 2x/p$$

Thus we can easily measure the wavelength of light emitted by a monochromatic source if we can count the number of fringes that appear in moving the mirror M2 through a distance x, which can be easily measured. The value of value of λ measured by a Michelson interferometer that appears in moving is very accurate since x can be measured by an accuracy of 10^{-7}m.

b) Determination of fine structure of spectral lines:

When a source of light emits closely spaced spectral lines, such as sodium doublet, having wavelength λ_1 and λ_2 each wavelength produces its own system of rings. Suppose $\lambda 1$ is only slightly greater than λ_2. Then for a small thickness film, the fringes correspond to these wavelengths will almost coincide in the entire field of view. But if mirror M_2 is moved away from splitter plate P_1, the fringes due to λ_1 and λ_2 begins to separate out. For a particular thickness of the air film, the dark fringes due to $\lambda 1$ will coincide with bright fringes due to λ_2 and become indistinguishable again. Moving mirror M_2 farther away will, however, make them distinct. Suppose that movement of mirror M_2 through a distance x makes n dark fringes due to λ_1 and (n+1) bright fringes due to λ_2 to appear at the centre. Then, we can write

$$x = n\,\lambda_1/2 = (n+1)\,\lambda_2/2$$

or $$n = 2x/\lambda_1$$

and $$n+1 = 2x/\lambda_2$$

so that $$\frac{2x}{\lambda_2} - \frac{2x}{\lambda_1} = 1$$

Hence, the difference in wavelengths is given by

$$\lambda_1 - \lambda_2 = \frac{\lambda_1 \lambda_2}{2x} \qquad (1)$$

If $\lambda_1 = \lambda_2$, we can replace the numerator on RHS of equation (1) by λ^2, where λ is the mean of λ_1 and λ_2. Therefore,

$$\lambda_1 - \lambda_2 = \frac{\lambda^2}{2x}$$

This result shows that once we measure the distance moved by the movable mirror between c=two consecutive positions of disappearance of the fringe pattern in the field view and know the mean wavelength λ, we can easily determine the difference between the two wavelengths.

c) **Determination of the refractive index of a thin film**

the refractive index of thin transparent plate of known thickness h can be measured by introducing it in the fixed arm of the interferometer. This will increase the optical path of this beam by $(\mu-1)h$. Moreover insertion of the plate produces a discontinuous shift in the fringe pattern and the number of fringes that cross the field of view cannot be counted. In fact if we use a monochromatic light source, it is impossible to identify as to which fringe in the displaced set corresponds to one in the original set. For this reason the interferometer is set to see straight fringes with monochromatic light and white light simultaneously before the plate is inserted. We first focus the cross-wire of the achromatic fringe. Then, the given plate is inserted in the path of one of the interfering waves. Since the wave traverses the plate twice, an extra path difference of $2(\mu-1)h$ is introduced between the two interfering beams. As a result, the fringe pattern gets shifted. Therefore, the movable mirror M_2 is moves till the fringes are brought back to their initial positions and the achromatic fringe is made to coincide with the cross-wires. If the distance moved by the mirror M_2 is x, we can write

$$2x = 2(\mu-1)h$$

or $$\mu = 1 + x/h$$

Alternatively, if p fringes cross the field of view, we can write

$$2(\mu-1)h = p\lambda$$

or

$$\mu = 1+p\lambda/2h$$

This result shows that once we know p, h and λ, we can easily determine the refractive index of the material of the plate. Alternatively, if we know μ, we can determine the thickness of the plate of very precisely.